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LETTER TO THE EDITOR

The superconductor–insulator transition in a Josephson junction chain with quantum fluctuation

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Abstract. The superconductor–insulator transition in a Josephson junction chain with quantum fluctuation is studied by a density matrix renormalization group. The critical Josephson coupling is found to be $J_c \sim 1.85$, while the Villain approximation gives $J_c \sim 1.1$. When extending the phase space from $(0, 2\pi)$ to $(0, 4\pi)$, corresponding to a very small quasi-particle tunnelling effect, the superconductor–insulator transition still exists but at a larger critical coupling $J_c \sim 2.3$.

The problem of the superconductor–insulator (SI) transition in a Josephson junction chain at $T = 0$ has been studied recently [1–6], but there are some controversies. The models considered are (A) the Josephson coupling between superconducting islands with charging energies (quantum fluctuation) and (B) with additional dissipation due to quasi-particle tunnelling or ohmic dissipation.

Case (A) was studied by Bradley and Doniach [1] by a perturbation theory leading to a Villain action, with the island phase space being considered as 2π periodic, corresponding to the quantization of island charges in the unit of Cooper pair charge, $2e$, where e is the electronic charge. The SI transition is predicted to be at $J_c \sim 2.47$ (see (1) below). Kampf and Schön [2] and Chakravarty *et al* [3] studied case (B) by the self-consistent harmonic approximation (SCHA) or the variational method with the phase space being either $-\infty < \phi < \infty$ (ohmic) or 4π periodic (quasi-particle tunnelling) corresponding, respectively, to continuous or integer multiple of e charge states on islands. The obtained phase diagram is characterized by two features. One is an SI transition at $J_c \sim 0.55$ for both ohmic and quasi-particle tunnelling in the weak-dissipation limit, and the other is an SI transition at a critical normal state resistance in the weak-Josephson-coupling limit. In the ohmic case, the SI transition in the weak-Josephson-coupling limit is claimed to be universal, the critical normal state resistance being $h/4e^2$ where h is the Planck constant. Model (B) with ohmic dissipation was also studied by Korshunov [4] by the instanton method. The universal critical resistance in the weak-Josephson-coupling limit was reproduced, and the SI transition predicted at $J_c = 8/\pi^2$ in the weak-dissipation limit, but an additional phase transition was predicted at half the critical normal resistance for J of order unity. This last transition was later extended by Bobbert *et al* [5] using a Villain approximation to the $J \gg 1$ region and argued to be a dipole (instanton–anti-instanton pair) to quadrupole transition, separating two superconducting phases of different properties. The latter authors found the SI transition in the weak-dissipation limit also at $J_c = 8/\pi^2$. Finally, Panyukov and Zaikin [6] studied model (B) by mapping it on a Ginzburg–Landau action using the Doniach procedure [7]. Among other things, they argued that there is no SI transition in the weak-dissipation limit in contradiction to the previous results. The universal SI transition in the weak-Josephson-coupling limit was also replaced by a resistive to resistive transition. Thus the problem is still open theoretically.

In this letter, we focus on one of the controversial points, namely the possible SI transition in the weak-dissipation limit. We thus consider the Hamiltonian

$$H = -\sum_i \frac{\partial^2}{\partial \phi_i^2} + J \sum_{\langle ij \rangle} \big(1 - \cos(\phi_i - \phi_j)\big) \tag{1}$$

where the first term is the charging energy (the self-capacitance model) and the second term describes the nearest-neighbour Josephson coupling. Using a density matrix renormalization group (DMRG) [8], we obtain the following results. (1) With the $(0, 2\pi)$ phase space, there exists an SI transition at $J_c \sim 1.85$. The Villain approximation also gives the SI transition but at a reduced value $J_c \sim 1.1$. (2) With the $(0, 4\pi)$ phase space, the SI transition continues to exist but at a larger coupling $J_c \sim 2.3$. The Villain approximation is a fairly routinely used procedure in statistical mechanics and condensed matter physics [9]. Our result (1) shows that this approximation is not quite accurate. Our result (2) agrees with those of the SCHA and variational method. The SCHA and the variational method, however, grossly underestimate the quantum fluctuation under the influence of small quasi-particle tunnelling. While we claim that these critical coupling constants are exact within ± 0.05, a quantum Monte Carlo calculation may be desirable to confirm the claim.

According to the phenomenological renormalization group theory [10], the Berezinskii–Kosterlitz–Thouless (BKT) transition [11] describing the SI transition under consideration can be determined by calculating the product of the system size, L, and the gap energy = the inverse correlation length, gap(L), as a function of the system size and the Josephson coupling J. To obtain gap(L), we calculate the ground state and the first excited states of the Hamiltonian (1) by the standard DMRG procedure [8]. We use the infinite algorithm, open-boundary condition, and the ground state target.

For the $(0, 2\pi)$ phase space, the basis states at each island are chosen to be the eigenstates,

$$\exp(\mathrm{i}n\phi) \qquad n = 0,\ \pm 1,\ \pm 2,\ \ldots \tag{2}$$

of the charge operator

$$-\mathrm{i}\frac{\partial}{\partial \phi} \tag{3}$$

where n represents the number of excess Cooper pairs. We truncate n to be $n = 0, \pm 1, \pm 2$, so the degree of local freedom is $m = 5$. The inclusion of the higher charge states gives a negligibly small correction to the result below. Our superblock size is $M = 40$. We checked the $M = 50$ case with negligible corrections to our result below. So for each system size, we have the four-block structure 40–5–5–40. We have also found that the three-block structure 40–5–40 works equally well. Figure 1 shows $L\,\mathrm{gap}(L)$ against J. The data collapsing at $J_c \sim 1.85$ indicates the BKT transition.

We have also calculated the same quantity for the Villain approximation which replaces the cosine potential by [9]

$$\mathrm{e}^{J\cos\phi} \sim I_0(J)\sqrt{2\pi J} \sum_{p=-\infty}^{\infty} \mathrm{e}^{-\frac{J}{2}(\phi - 2\pi p)^2} \equiv \mathrm{e}^{JV(\phi, J)} \tag{4}$$

where $I_0(J)$ is the zeroth-order modified Bessel function. Note that the Villain potential $V(\phi, J)$ depends on J, and the approximation improves for a larger J. Figure 2 shows

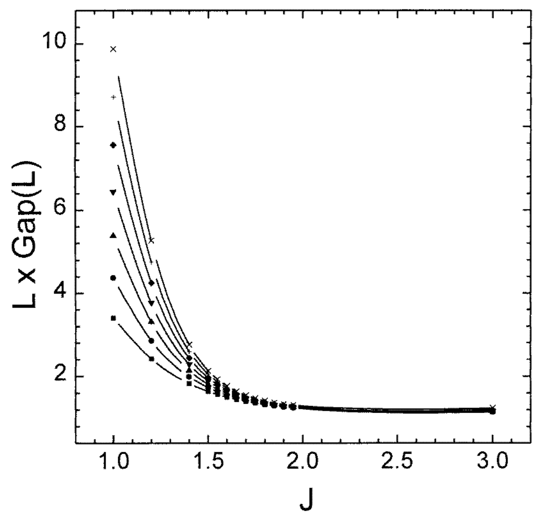

Figure 1. L gap(L) against J for the (0, 2π) phase space. The lines are from the bottom to top for the system sizes $L = 13$, 19, 25, 31, 37, 43 and 49.

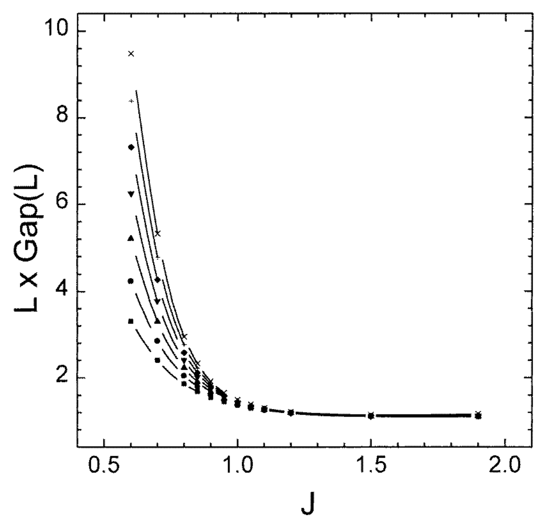

Figure 2. The same as in figure 1 but for the Villain approximation.

L gap(L) against J for the Villain approximation. The data collapsing at $J_c \sim 1.1$ indicates the BKT transition. Note the underestimation by a factor of about two of the critical Josephson coupling J_c in the Villain approximation. When considering a delicate issue such as the universal critical resistance in superconducting granular films or two-dimensional Josephson junction arrays [12], the Villain approximation must be used with caution.

Turning to the (0, 4π) phase space case, the basis states at each island are chosen to be the eigenstates

$$\exp(in\phi) \qquad n = 0 \pm \tfrac{1}{2},\ \pm\tfrac{2}{2},\ \pm\tfrac{3}{2},\ \ldots \tag{5}$$

i.e. the island charge is now allowed in integer multiples of the electronic charge due to

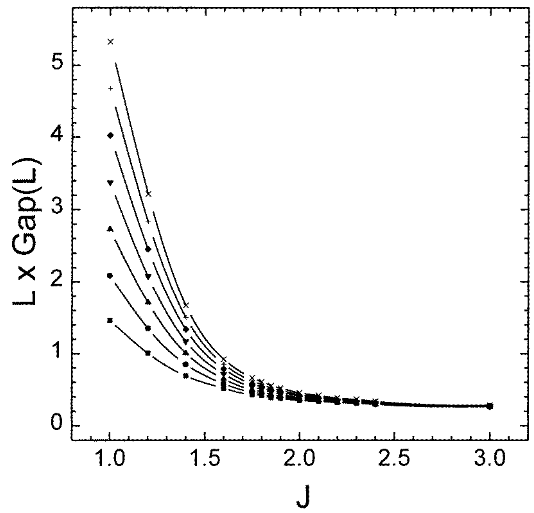

Figure 3. The same as in figure 1 but for the $(0, 4\pi)$ phase space.

the tunnelling of quasi-particles. Here we are interested in the limit of small quasi-particle tunnelling, thereby neglecting the action which describes the quasi-particle tunnelling (see, e.g., (8) in [4]), and only taking into account the phase space enlargement effect. We took the first 15 eigenstates, thus the elementary block size is $m = 15$. $m = 17$ was tested in some cases, giving a negligible correction. The superblock size is again $M = 40$, with $M = 50$ being tested to give negligible correction. It is noted that m cannot be as small as in the $(0, 2\pi)$ phase space case. Accordingly we have to choose the three-block structure instead of the four-block structure from a computational restriction. From the general idea of DMRG, one might expect a rather poor performance for the three-block algorithm. As mentioned above, however, we have found that the three-block structure works equally well. We have further tested the accuracy of the three-block algorithm in a similar system of coupled harmonic oscillators which is soluble by hand [13]. Figure 3 shows $L\,\mathrm{gap}(L)$ against J indicating the SI transition at $J_c \sim 2.3$. This result agrees with those of the SCHA and the variational method [2, 3]. These methods, however, grossly underestimate the quantum fluctuation under the influence of small quasi-particle tunnelling.

Finally a comment is due on the ohmic dissipation case. Experimentally, there is always a small current leakage and thus dissipation. Nevertheless the SI transition was clearly observed in a single Josephson junction and the same appears to be the case for a chain [14]. Computationally, the case requires one order more expensive calculations than the $(0, 2\pi)$ and $(0, 4\pi)$ cases, roughly the same amount of effort as to evaluate precisely the path integral expression of the original partition function for finite ohmic dissipations. Theoretically, however, the phase space issue, i.e., whether it is 2π periodic, 4π periodic or not periodic at all, corresponding respectively to the $2e$ discrete, the e discrete and continuous charges on each island, is not quite clear [15]. In fact, it is rather conceptually difficult to imagine continuous charge states in light of the fact that any electronic processes taking place in a combined system of a Josephson junction array and its environment strictly conserves the quantization of electronic charges. The phase space issue must be resolved in order for us to make further progress in the understanding of the Josephson junction arrays.

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